# 18. VERIFICATION OF THE TYCHO CATALOGUE: ASTROMETRY

*The Tycho astrometry of stars is verified through comparisons with the Hipparcos Catalogue and the Catalogue of Positions and Proper Motions (PPM). Systematic errors are negligible, of the order 1 mas and 1 mas/yr. The external standard errors, on the other hand, exceed the formal standard errors given in the Tycho Catalogue for stars fainter than $V_T \simeq 9.5$ mag; at $V_T = 10.5$ mag by a factor 1.5. Observations of solar system objects are compared with Hipparcos and ground-based results in Chapter 15.*

## 18.1. Astrometric Standard Errors

Internal and external standard errors for the Tycho Catalogue are discussed in this section. There are two kinds of internal standard errors: the 'internal formal' standard error which is derived from the diagonal elements of the covariance matrix of the least-squares solution, and the 'internal corrected' standard error which is obtained from the internal formal standard error by multiplication with the unit-weight error, derived from the residuals, see Section 11.1. Finally, the 'external' standard error can only be estimated by comparison with other catalogues of compatible or superior quality.

In this terminology all standard errors given for photometry and astrometry of the individual stars in the Hipparcos and Tycho Catalogues are internal corrected standard errors, except for the astrometric values in the Hipparcos Catalogue where internal formal standard errors are given. The background for the choice of internal corrected standard errors for the astrometric quantities in the Tycho Catalogue shall be given.

The internal formal standard errors for a given set of accepted observations depend only on the way the weighting of individual observation equations has been done, see Section 7.1. The weights of observations in the Tycho reduction were inversely proportional to the square of the expected standard error of the transit time $\sigma_u$ (also called the *a priori* error) in Equation 7.6. The latter comprised the photon-statistical error $\sigma_\tau$, which was calculated as a function of only the estimated amplitude and background for a given transit, based on an advanced theory of Maximum Likelihood (ML) estimator for the astrometric location problem by Yoshizawa *et al.* (1985). It had been proven in the same paper by numerical simulations, that the non-linear ML technique and the linear digital filtering technique, which was in fact used in the Tycho reduction algorithms, had very

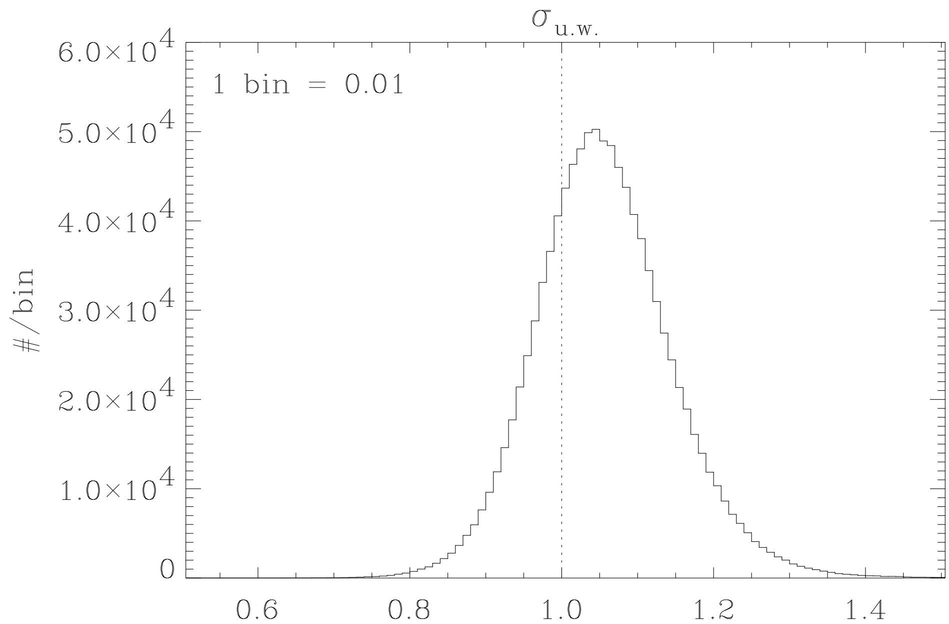


***Figure 18.1.*** *Distribution of the unit-weight error for the stars in the Tycho Catalogue.*

similar performances at magnitudes up to 10. It should be noted that the Tycho filtering (see Chapter 2) was in fact non-linear only in the presence of spikes or if bright stars disturbed each other. Otherwise it was linear and thus ensured the quick processing.

At fainter magnitudes than 10, however, the difference between the two methods could be significant, but the results of the simulations were rather uncertain. For the lack of something better, the strictly theoretical expression of the Maximum Likelihood estimator was used as *a priori* errors. It was checked several times in the course of the data reduction, that the error model provided a quite reliable result for stars brighter than 10 mag, while for the fainter majority of stars the observed scatter of transit times typically exceeded the *a priori* error. This can be seen in Figure 18.1, where the unit-weight error distribution is shown for the stars in the Tycho Catalogue. The distribution is centered at 1.05, meaning that the formal error was typically 5 per cent smaller than the observed scatter. If such formal errors were used as standard errors in the Tycho Catalogue, as was done in the Hipparcos Catalogue, then the final precision of the astrometric parameters would have been systematically overestimated by merely 5 per cent on average, but it could be larger than 20 per cent for many stars (Figure 18.1).

There are several reasons for the deviation of the unit-weight error, $\sigma_{\mathrm{u.w.}}$, from unity. First, it is expected on theoretical grounds that the Maximum Likelihood estimator theory does not apply well to extremely faint signals, which correspond to about magnitude 11 for Tycho. Even if it did, the linear filtering in detection could have performed in a different way at faint magnitudes, yielding a slightly worse estimation. Second, there were some features of the Tycho astrometric observations, which could not be properly accounted for by a simple and practical error model. The disturbing influence of unrecorded parasitic signals was perhaps especially strong on faint target stars. Besides, this influence could be different for different stars, depending on the neighbourhood, possible duplicity, etc. This was a good reason to choose the correction of the standard error by $\sigma_{\mathrm{u.w.}}$ for each star individually, as stated in Section 11.1, rather than trying to find an empirical error model, fitting the real uncertainties of observations at faint magnitudes.

The unit-weight error can be calculated for every star in the Tycho Catalogue, given a goodness-of-fit ($F2$) value, by means of the equation referred to in Volume 1, Section 2.2 (Field T30). A few stars have no $F2$ value and are excluded from this discussion. The bias of the $\sigma_{\text{u.w.}}$ distribution is correlated with various quality measures in the Tycho Catalogue. For instance, the mean $\sigma_{\text{u.w.}}$ for non-recommended astrometric stars (Field T10 = 'X') is 1.11, but it is as small as 1.04 for the recommended reference stars. There is also a systematic variation of this statistics with $V_T$ magnitude, see Figure 18.2. The unit-weight error appears to depend on the number of astrometric observations, $N_{\text{astrom}}$, for stars brighter than $V_T = 10.0$ mag, but not for fainter stars. The reason for the dependence might be that a relatively smaller number of observations for a bright star will remain when a larger than average number of observations are rejected due to parasitic transits, e.g. from the other component in a double star. It is noted that there are 84 000 stars in the Tycho Catalogue with $N_{\text{astrom}} < 70$, and about 16 000 of them are brighter than 10.0 mag. A representative histogram of $N_{\text{astrom}}$ is given in Figure 7.6(d).

Since the unit-weight error is generally not larger than 1.10, it is concluded that the internal model of statistical weights was satisfactory. It will, however, appear from the following analysis of external errors of the Tycho Catalogue, that although the correction has in general increased the standard errors in the catalogue, and for faint stars particularly so, it was not sufficient to provide a quite correct measure of astrometric accuracy, at faint magnitudes.

The ratio of external standard errors to internal corrected standard errors is satisfactory, about 1.1 for the five astrometric parameters for stars with $V_T < 10$ mag. It is about 1.5 at the median magnitude $V_T = 10.5$ mag for positions (Figure 18.4), for proper motions (Figures 18.8(d) and 18.8(e)), and for parallaxes (Figure 18.9). The ratio increases to 1.8 at $V_T = 11.0$ mag, and the main reason is probably the rejection of outliers (Chapter 7) which is quite significant at faint magnitudes.

## 18.2. Comparison with the Main Hipparcos Catalogue

The Hipparcos Catalogue has been used for deriving calibration parameters for the Tycho Catalogue and for defining the system of Tycho positions, parallaxes and proper motions. This section and Section 18.3 show to which extent the Hipparcos and Tycho systems agree and to which extent the error estimates in the Tycho Catalogue are realistic. In Sections 18.3–18.5 properties of positions, proper motions and parallaxes will be treated separately.

Figure 18.3 shows sky plots of the median residuals, 'Hipparcos–Tycho', in cells of $15° \times 15°$. Residuals were computed at the Tycho mean observational epoch for the 68 103 recommended reference stars in common and brighter than $V_T = 9$ mag. Each cell therefore represents about 370 stars. The values are close to zero, as expected, with a cell-to-cell scatter of 0.3 mas for positional coordinates, 0.4 mas for parallax and 0.7 and 0.4 mas/yr for the two proper motion components. With 370 stars in each cell and median standard errors of 5–7 mas for the Tycho Catalogue stars a cell-to-cell scatter of 0.3 mas is expected. Except for the proper motion in right ascension, this is very close

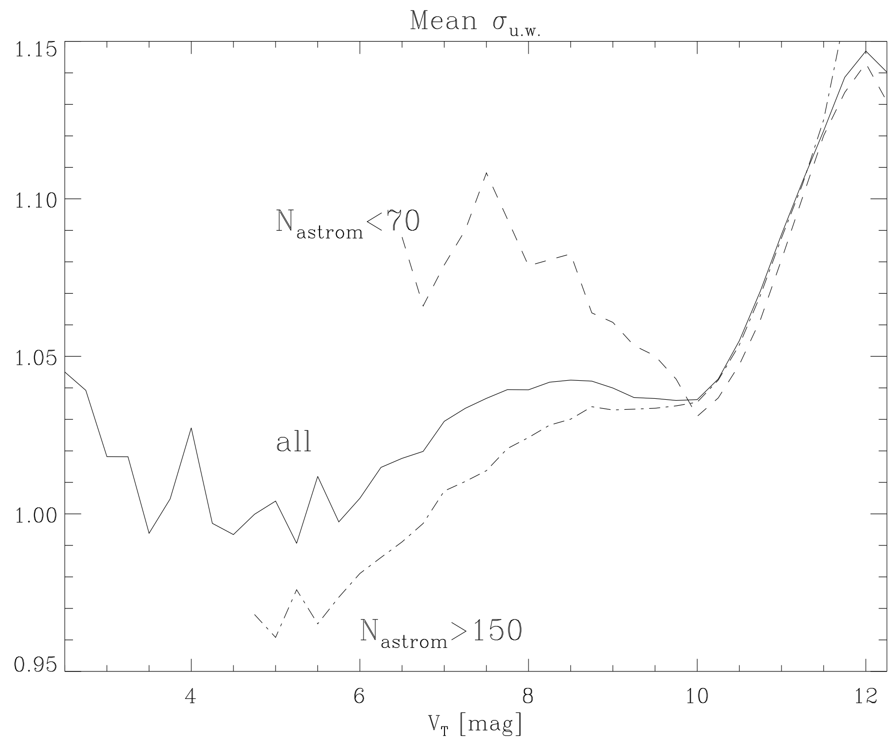


***Figure 18.2.*** *Mean unit-weight error, $\sigma_{u.w.}$, versus $V_T$ magnitude for all stars in the Tycho Catalogue, and for stars with $N_{astrom}$ greater than 150 and smaller than 70, as indicated in the plot.*

to the actual values. We may therefore conclude that the Tycho system is practically identical to the Hipparcos system on a 15° scale.

In order to estimate the reliability of the internal standard errors given in the Tycho Catalogue, the residuals 'Hipparcos–Tycho' discussed above were also measured in units of the internal standard error of 'Hipparcos–Tycho'. Ideally, the distribution of this quantity should have a median of 0 and the 16th and 84th percentiles at −1 and +1, respectively. Figure 18.4 shows half the difference between the 84th and 16th percentiles as a function of $V_T$, the sample of stars having been extended also to include the fainter stars. This half difference can be understood as a robust estimate of the ratio between the external and the internal errors. Any deviation from the expected value of +1 can be attributed to the Tycho Catalogue because the errors of the Hipparcos Catalogue are negligible in this comparison, except for the brightest stars.

For the positional coordinates, the ratio between the external error and the internal standard error lies at 1.05 for stars brighter than $V_T = 9.25$ mag; it then rises rapidly for fainter stars, reaching 1.5 at $V_T = 10.5$ mag (the median magnitude for Tycho) and 1.8 at $V_T = 11$ mag. For parallax and proper motion the ratio stays close to 1.0 for all magnitudes. There is, however, strong evidence that the external errors in parallax and proper motion for non-Hipparcos stars follow if not the same then at least a similar pattern as the right ascension and declination. The difference between Hipparcos and non-Hipparcos stars arises because Hipparcos parallaxes and proper motions were used as starting values in each iteration of the astrometric reductions as explained in Section 11.1. The properties of non-Hipparcos stars are discussed further in Sections 18.3 and 18.6.

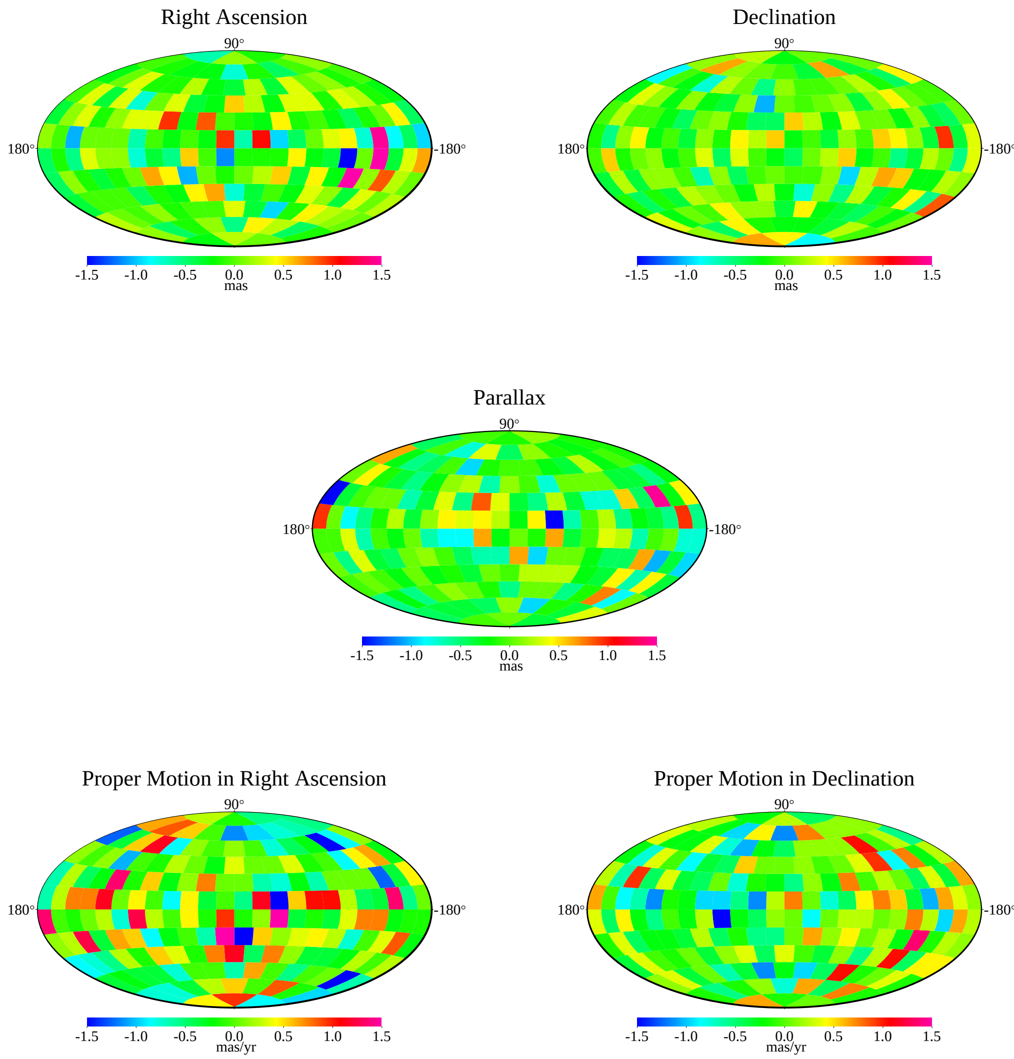


***Figure 18.3.*** *Median differences, 'Hipparcos–Tycho', in equatorial coordinates for recommended astrometric reference stars brighter than* $V_T = 9$ *mag. The cell size is* $15^\circ \times 15^\circ$.

The cross-identification between the Hipparcos and Tycho Catalogues was generally simple with more than 110 000 stars identified closer than 100 mas from the Hipparcos position. A general identification limit of 1 arcsec was adopted, but in a few cases identity was accepted at distances up to 1.8 arcsec. In such cases either Tycho or Hipparcos must be in error, or a photocentre of a double star system was cross-identified with a resolved component. Double stars are often resolved in the Hipparcos Catalogue but not in the Tycho Catalogue. Figure 18.5 shows the tail of the distribution of distances between the Hipparcos and Tycho positions for cross-identified catalogue entries of quality $Q = 1-8$ stars (see Table 7.1). Double stars are only included in the plot if resolved in both catalogues. This means that 97 665 single stars and 13 072 components of double stars are included. The tails beyond 200 mas, shown in the figure, contain only 95 (0.1 per cent) and 253 (1.9 per cent) stars, respectively. The double star distribution is somewhat broader than for single stars as could be expected. More significant, though very small, is the peak with 40 stars at 1.25 arcsec. Errors in the Tycho treatment of double stars

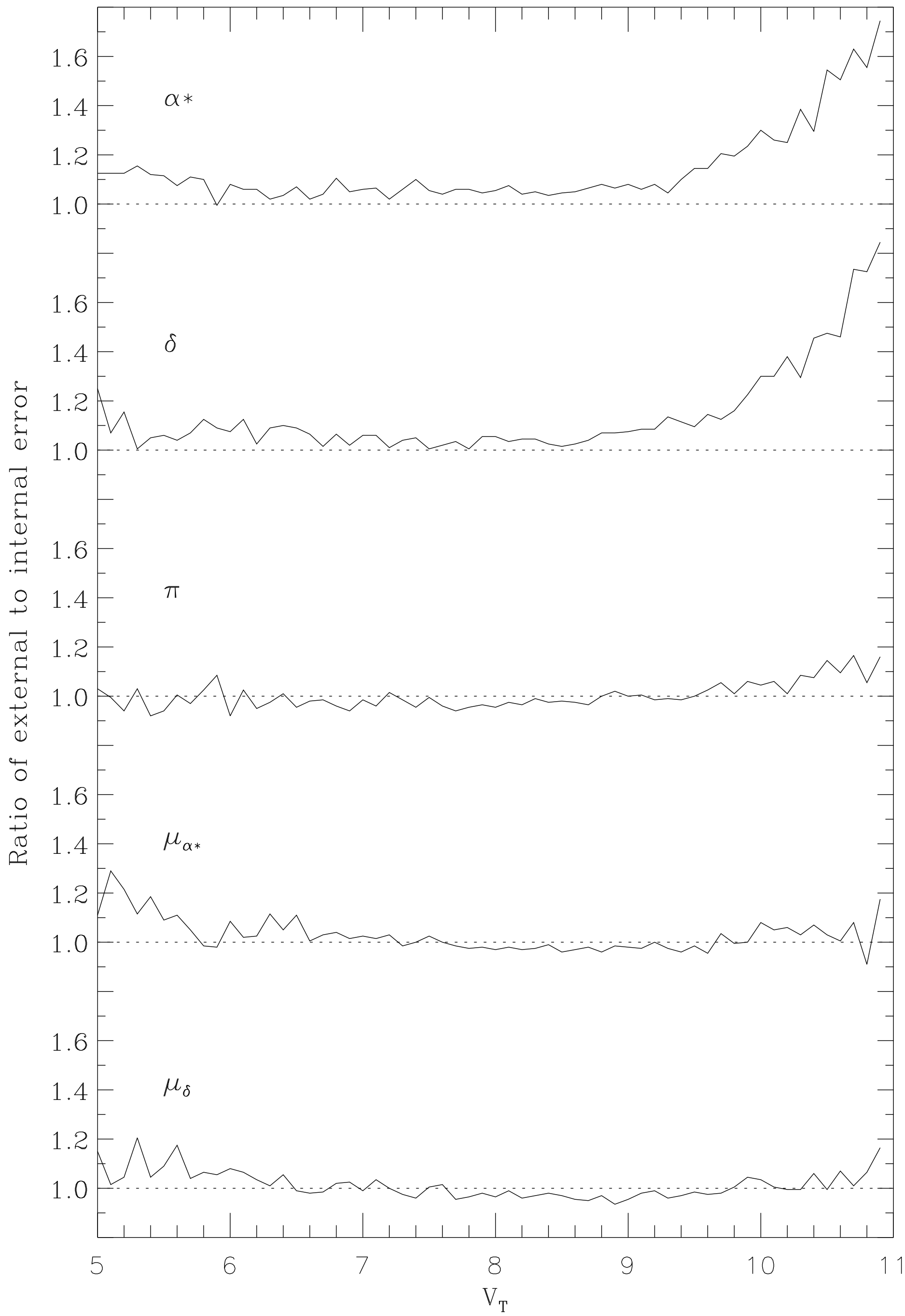


***Figure 18.4.*** *The ratio of external error to internal standard error for the Tycho Catalogue derived from a comparison with Hipparcos values. The absence of a rise for parallax and proper motion for stars fainter than 9 mag only applies to Hipparcos stars, as explained in the text.*

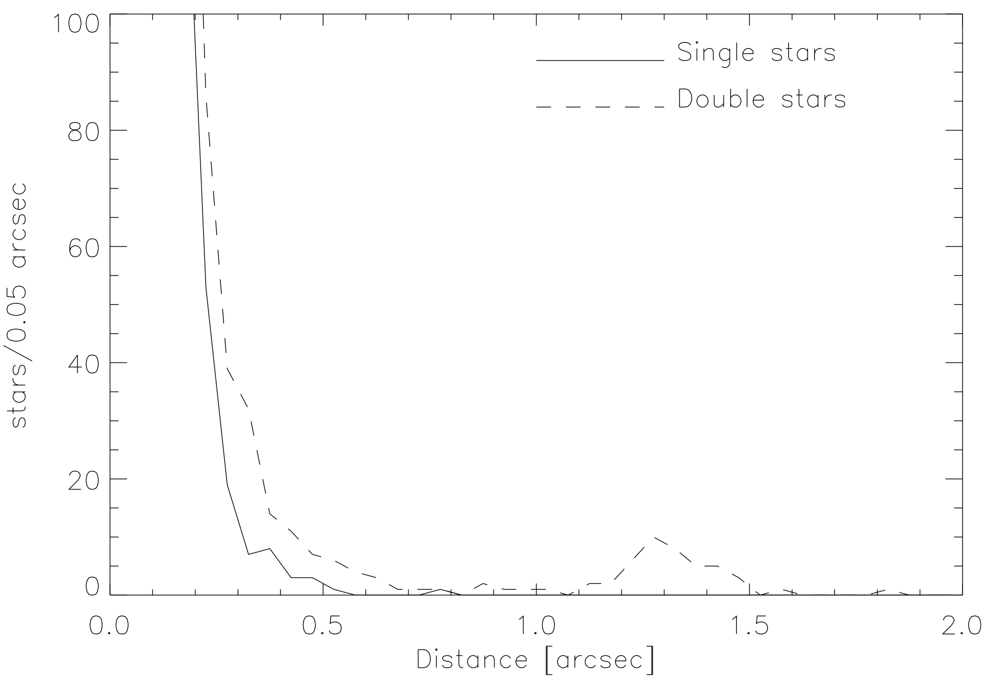


***Figure 18.5.*** *Tail of the distribution of distances between Hipparcos and Tycho positions for cross-identified single stars and resolved components. See the text for further explanation.*

can certainly not be excluded, but the prime suspect is in fact Hipparcos. The grid used in the main mission has a period of 1.21 arcsec. This means that the abscissa derived in a particular transit is only known modulo 1.21 arcsec. A subtle combination of input catalogue error, dominating scan directions and configuration of the double or multiple system, can result in such grid-step errors finding their way to the final catalogue. In the Hipparcos Catalogue, the quality of the solution is normally quite low in these cases, with standard errors in the range 4–120 mas for 78 per cent of the stars.

In the Hipparcos main mission, the instantaneous field of view has a diameter of 38 arcsec. All stars within this field contribute to the signal and will ideally be resolved in the double star treatment. We therefore do not expect to find Tycho stars near Hipparcos stars, unless the Tycho star is also a Hipparcos star. The stars that we do find lead us to suspect errors either in the Tycho Catalogue or in the Hipparcos Catalogue. Figure 18.6 shows the distribution of distances between Tycho stars not cross-identified in the Hipparcos Catalogue and Hipparcos stars not cross-identified in the Tycho Catalogue. The question is, if some of these pairs are artefacts caused by large errors in Hipparcos or Tycho. There are 64 pairs involving 59 quality $Q = 1$–$8$ Tycho stars and 58 pairs involving quality $Q = 9$ stars. Also plotted are the positions corresponding to grid-step errors in Hipparcos and side lobes in Tycho.

There are no pairs with small separations simply because such pairs are cross-identified. In Figure 18.6(a), representing the good Tycho positions, there is a strong correlation between the distribution of pairs and the likely error-induced distances for distances up

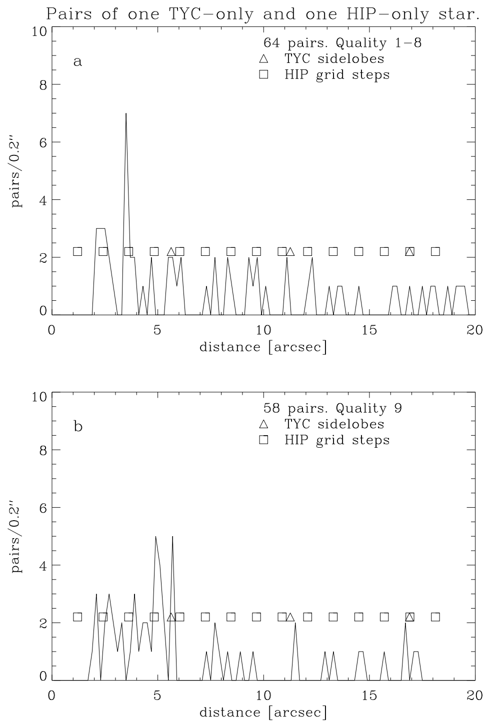


***Figure 18.6.*** *Distribution of the distances between entries in the Tycho and Hipparcos Catalogues with no cross-identification, called respectively TYC-only and HIP-only stars. The distances introduced by typical error sources are marked. (a) pairs with quality $Q = 1 - 8$; (b) pairs with $Q = 9$.*

to 6–7 arcsec, whereas the correlation is rather weak at higher distances. These quality $Q$ = 1–8 stars all belong to double or multiple systems, a fact leaving open the possibility that both Tycho and Hipparcos are correct albeit incomplete, but also the possibility that errors have indeed been introduced. In Figure 18.6(b), representing the low quality Tycho stars, the correlation is weak at all distances. The quality $Q = 9$ stars are, by definition, of low quality so these 58 pairs are probably dominated by errors in Tycho.

## 18.3. Comparison with the PPM Catalogue

A comparison with positions in the Catalogue of Positions and Proper Motions (PPM) (Röser & Bastian 1991; Bastian *et al.* 1993) is of limited value for the verification of the Tycho Catalogue. For the proper motions, however, a comparison is useful because the PPM proper motions have standard errors of only 4 mas/yr, compared to typically 30 mas/yr for the Tycho Catalogue.

An investigation of the PPM (Fabricius 1993), based on recent ground-based observations, has shown a magnitude equation in PPM and the comparison was therefore carried out for bright and faint stars separately. Figure 18.7 shows a comparison, 'TYC−PPM', for the four available astrometric parameters. There are indeed some striking differences between the stars brighter than $B_T = 10$ mag and the fainter stars. In right ascension, differences are prominent for faint stars around declination 20° at right ascension 0–10 hours. In declination there are interesting features for faint stars around declination

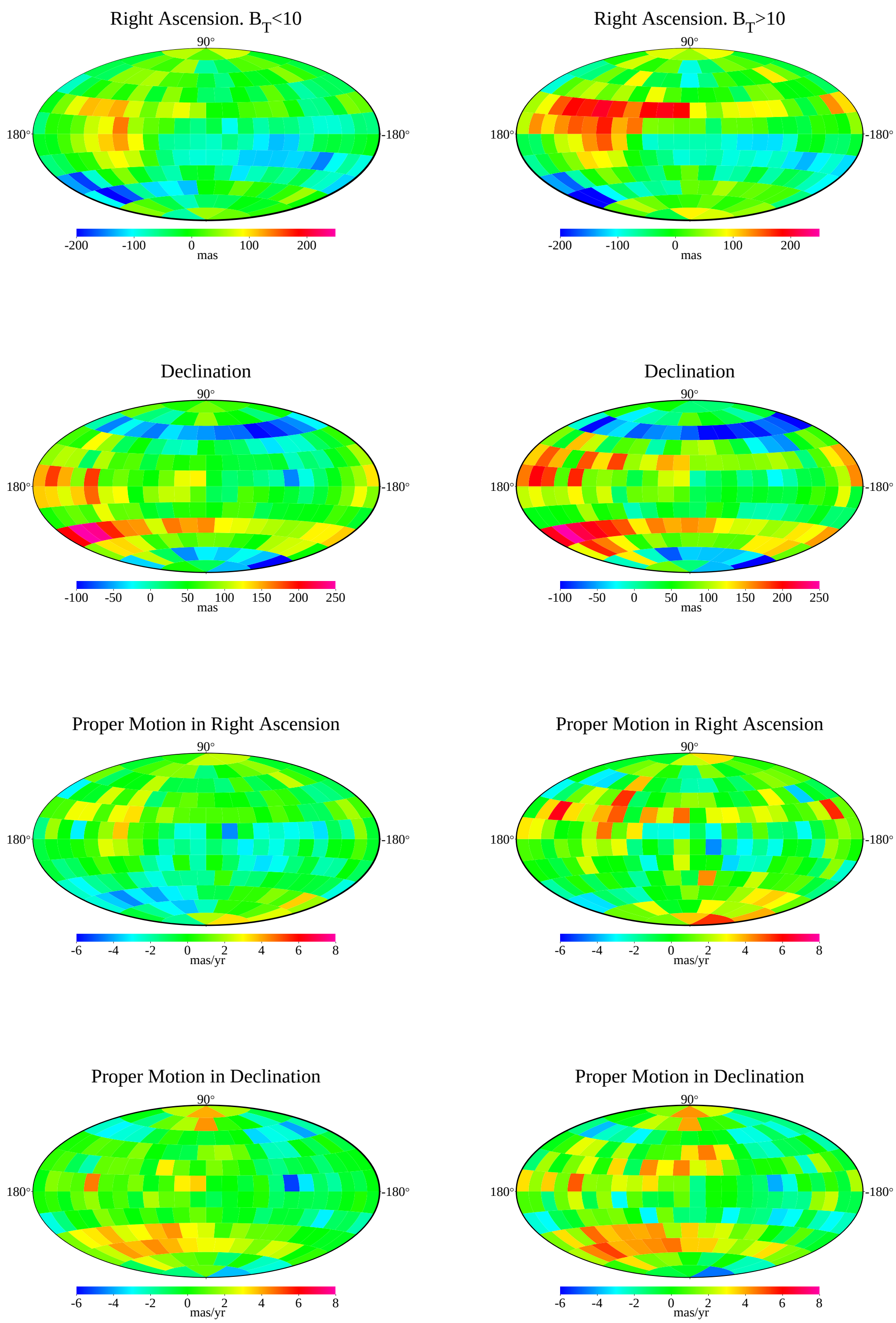


***Figure 18.7.*** *'TYC–PPM' in equatorial coordinates, where TYC is the Tycho Catalogue and PPM is the Catalogue of Positions and Proper Motions. The four figures to the left show the median difference for the 134 760 stars brighter than $B_T = 10$ mag and the four figures to the right show the median difference for the 276 737 stars fainter than $B_T = 10$ mag. The cell size is $15^\circ \times 15^\circ$.*

20° and 50°. In position the cell-to-cell scatter is around 70 mas, giving the order of magnitude of the systematic differences between PPM and ICRS. For proper motion the cell-to-cell scatter is around 2 mas/yr and values rarely exceed ±5 mas/yr. This is the same order as the internal standard error of the PPM Catalogue, and well below the internal standard error of the Tycho Catalogue proper motions for stars fainter than $V_T \simeq 9$ mag. It is therefore meaningful to make a comparison between the two catalogues for the proper motions of faint stars, without applying systematic corrections.

Figure 18.8 shows a comparison between proper motions in the PPM Catalogue and in the Tycho Catalogue. Figure 18.8(a) shows the median standard error in the Tycho Catalogue as a function of $V_T$. No difference can be seen between Hipparcos and non-Hipparcos stars. Figure 18.8(b) shows the much smaller standard error in the PPM Catalogue. Hipparcos stars have generally smaller errors than other stars probably because many Hipparcos stars, e.g. the IRS, are among the high precision subset of the PPM Catalogue. Figure 18.8(c) combines the internal errors of Tycho and Hipparcos, giving the internal standard error of 'TYC−PPM'. The distribution of differences in proper motion, 'TYC−PPM', measured in units of the internal error, has its median around 0 and the 16th and 84th percentiles around −1 and +1. The half difference between these two percentiles can be interpreted as the ratio between external and internal error. It is shown in Figures 18.8(d) and 18.8(e) for the two components of proper motion. For stars brighter than 9 mag the values are rather high because the systematic errors of the PPM Catalogue have been ignored, but these stars are not our main concern. At fainter than 9.5 mag the ratio lies at 1.05 for Hipparcos stars, but considerably higher for other stars with 1.25 as the minimum, reaching 1.5 at $V_T = 10.5$ mag and 1.8 at $V_T = 11$ mag. The latter two values are exactly the same as found for Tycho positions in the preceding section in the comparison with Hipparcos.

It is concluded that internal standard errors of Tycho proper motions need the same correctional factor as for positions. For brighter stars ($V_T \simeq 7 - 9.5$ mag) Figure 18.4 shows that the ratio of external to internal error is in fact somewhat less than 1.0 for Hipparcos stars. If we now normalize Figures 18.8(d) and 18.8(e) to give the same ratios for Hipparcos stars as in Figure 18.4, the curves for non-Hipparcos stars will, of course, still lie 10 per cent higher than for Hipparcos stars, but now at a level of 1.05–1.1, which is again comparable to what was found for Tycho positions.

## 18.4. Tycho Positions

It was demonstrated in Section 18.2, through comparisons with the Hipparcos Catalogue, that the Tycho and Hipparcos systems are practically identical. It was also shown that the standard errors in the Tycho Catalogue are too small for stars fainter than $V_T \simeq 9.5$ mag. At $V_T = 10.5$ mag a factor of 1.5 should be applied and at $V_T = 11$ mag the factor is 1.8.

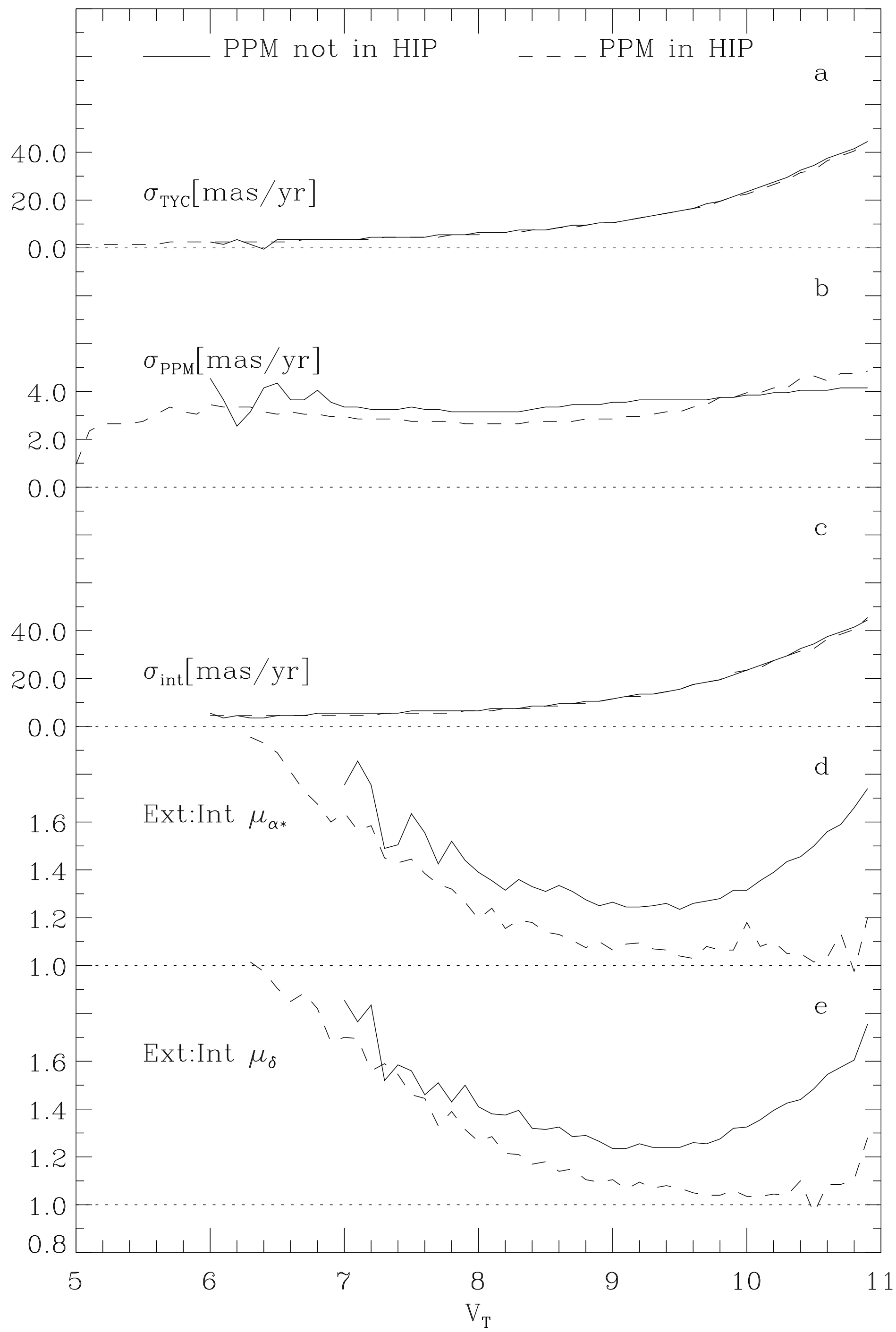


***Figure 18.8.*** *Median internal standard error of (a) the Tycho Catalogue proper motions, (b) of proper motions from the Catalogue of Positions and Proper Motions (PPM), and (c) of their difference. The ratio between external and internal errors of the Tycho Catalogue proper motions (d and e), see text. The high ratios for bright stars are only due to the rather simple method used for comparison, as explained in the text.*

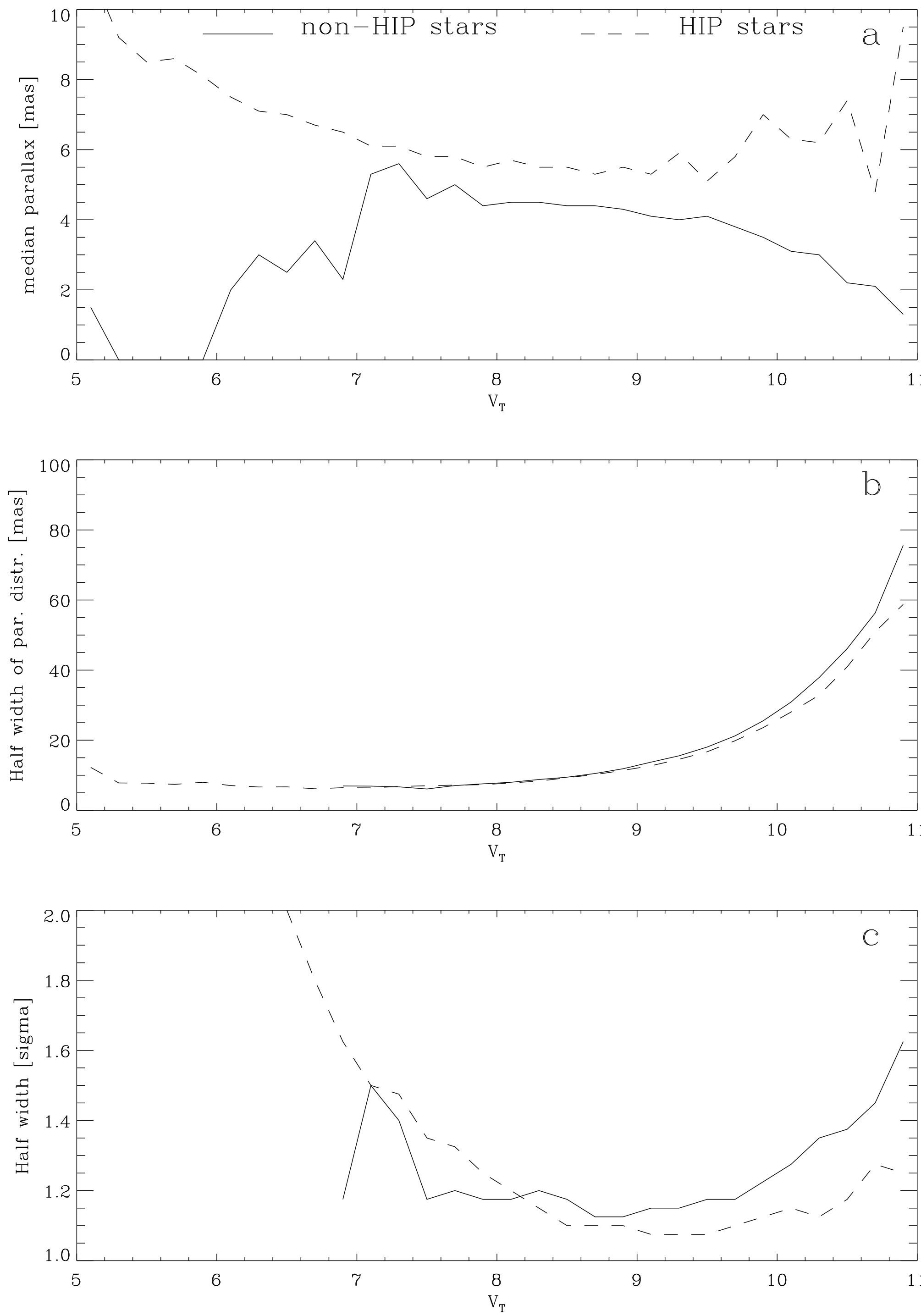


***Figure 18.9.*** *The median Tycho parallax for Hipparcos and non-Hipparcos stars is shown in (a). The half width of the parallax distribution, derived from the 16th and 84th percentiles, is shown in units of mas in (b) and in units of the internal standard error in (c). When the parallax is much smaller than the error, the curves in (c) represent the ratio between external and internal error. This assumption does not hold for the bright stars resulting in the steep rise. See the text for further explanation.*

## 18.5. Tycho Proper Motions

It was shown in Section 18.2 that the Tycho proper motions agree systematically with the Hipparcos proper motion. The proper motions were also addressed in Section 18.3 in a comparison with the Catalogue of Positions and Proper Motions (PPM). The standard errors of Tycho proper motions were shown to need the same correction as for positions, i.e. no correction for stars brighter than $V_T \simeq 9.5$ mag, at $V_T = 10.5$ mag a factor of 1.5 should be applied and at $V_T = 11$ mag the factor is 1.8.

## 18.6. Tycho Parallaxes

In the Tycho Catalogue there are 1132 non-Hipparcos stars with relative parallax errors less than 20 per cent, but in general the parallaxes are of low significance for individual stars. For larger samples, the Tycho parallaxes can be of value. There are, for example, more than 38 000 non-Hipparcos stars in the Tycho Catalogue with standard errors below 10 mas.

In Section 18.2 it was shown that the Tycho parallaxes agree with the Hipparcos parallaxes on a global scale. It was shown in Sections 18.2 and 18.3 that the internal standard errors for position and proper motion of non-Hipparcos stars in Tycho have been underestimated for stars fainter than $V_T = 9$ mag. In order to test if the same is true for parallaxes we note that for faint stars, the standard error is generally much larger than the parallax. We can therefore assume that the true parallax is zero and get a conservative estimate of the external error. Figure 18.9 shows the median parallax (Figure 18.9(a)) which is much smaller than the half width of the distribution of parallaxes (Figure 18.9(b)) for the faint stars. The half width has been computed as the half difference between the 16th and 84th percentiles. Figure 18.9(c) shows the half width of the distribution of the parallax divided by the internal standard error. At the bright end the curve rises because the assumption of zero true parallax does not hold. At the faint end we find a ratio of external to internal error of 1.4 at $V_T = 10.5$ mag and 1.7 at $V_T = 11$ mag. These values are slightly smaller than what was found for position and proper motion.

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