12. EPHEMERIDES, TIMING, AND CALCULATION OF CELESTIAL DIRECTIONS

The interpretation of the observations in terms of astrometric parameters of stars or the motions of solar system objects required the use of auxiliary information in the form of ephemerides of the satellite and other bodies; the use of a single, uniform time scale that could be related to celestial phenomena; and the use of precise mathematical models for the calculation of celestial directions. This chapter describes the implementation of these utilities by the reduction consortia, and summarises the results of tests to compare the implementations.

12.1. Ephemerides

The ephemerides used in the Hipparcos data reductions describe the time-dependent relative positions of five points in space: the solar system barycentre, the Earth, the Sun, the observer (in this case the Hipparcos satellite) and the observed object, which may be a solar-system object or a star. In the latter case, the astrometric parameters of the star may be regarded as defining the ephemeris of its barycentric motion. The vector relations between the five points are illustrated in Figure 12.1. Each vector is a function of time T, for which the Terrestrial Time (TT) was used throughout; the periodic differences (of up to ± 1.6 mas) with respect to a barycentric coordinate time scale were thus neglected in the calculations. For the computation of occultations, an approximate ephemeris of the Moon was also needed.

Earth and Moon

The ephemeris of the Earth, $\mathbf{b}_{\mathrm{E}}(T)$, was supplied by J. Chapront, G. Francou and B. Morando from the Bureau des Longitudes in Paris. It consisted of the ephemeris of the barycentre of the Earth–Moon system with respect to the solar system barycentre on one hand, and of an ephemeris of the Earth relative to the Earth–Moon barycentre on the other. Regarding the former, a compact representation of the position and velocity vectors was derived from the best analytical theories to the required precision of 10 km in position and 0.05 m s^{-1} in velocity, in the form of a Fourier-Poisson expansion over intervals of 400 days each.

The motion of the Earth with respect to the Earth–Moon barycentre was given by Fourier expansions based on the planetary and lunar theories developed at the Bureau

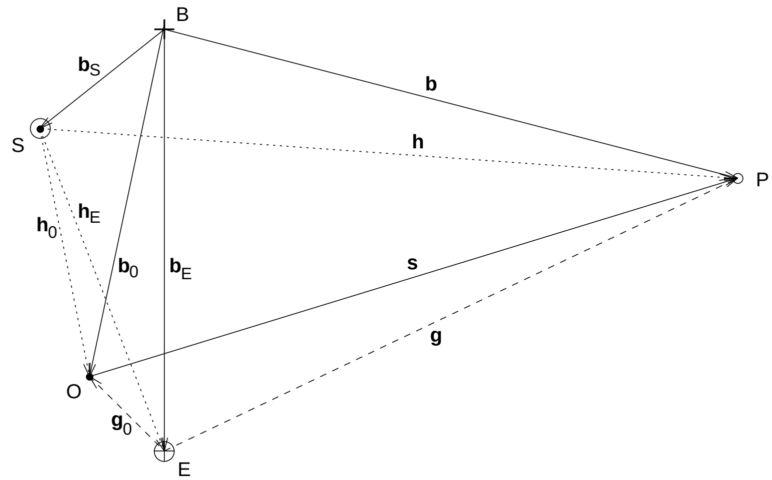

Figure 12.1. *Vector relations between the solar system barycentre (B), the Sun (S), the Earth (E), the observer (O) and the point under observation (P). Barycentric, heliocentric, geocentric and topocentric vectors are denoted* **b**, **h**, **g**, *and* **s**, *respectively, with indices as appropriate for the different bodies.*

des Longitudes. There were 22 harmonics for the ecliptic X and Y coordinates and their time derivatives, and eight for the ecliptic Z and $\mathrm{d}Z/\mathrm{d}T$. An approximate position of the Moon, used to calculate occultations, was derived from the same data. The ecliptic Earth ephemeris was converted to equatorial coordinates using the IAU value for the obliquity of the ecliptic at J2000, see Table 12.1.

A tabulation of the barycentric equatorial position and velocity components of the Earth, as calculated from the subroutines used in the reductions, is given in Table 12.2.

It can be noted that the reduction consortia interpreted this ephemeris as applying to the provisional reference frame of their respective solution, or actually the solution of the previous iteration; no attempt was made to take into account the difference between that frame and the actual reference frame of the ephemeris (nominally the dynamical system for the equinox of J2000). Thus, for the final FAST and NDAC solutions, the ephemeris was assumed to refer to the reference frames of F37.1 and N37.1, respectively. The orientations of these frames relative to the Hipparcos Catalogue (or the ICRS system) are given in Table 16.8.

Sun

The ephemeris of the Sun, $\mathbf{b}_S(T)$, was needed for calculations of the gravitational light deflection (Section 12.3) and for eclipse calculations. A simple elliptical motion was assumed by FAST, while NDAC used a polynomial fit to the barycentric ephemeris of the Sun derived by Clemence (1953), but modified to modern values for the masses of Pluto and Saturn (respectively 1/130 000 000 and 1/3498.5 of the solar mass).

Minor Planets and Other Solar System Objects

The observing programme included some 60 minor planets (of which 48 were actually observed), two moons of Saturn and one moon of Jupiter. Geocentric ephemerides for these objects, $\mathbf{g}(T)$, were supplied by the Bureau des Longitudes in Paris. For the minor planets the data (ecliptic longitude and latitude, distance and magnitude) were given in the form of Chebyshev polynomials of order 8; for the moons as trigonometric series for the positions relative to the ephemerides of the parent planets, which were given as Chebyshev polynomials of order 10. The distances to the moons were taken equal to the distance of the parent planet. The geocentric position vector resulting from these calculations included a correction for planetary aberration. A transformation to equatorial coordinates was made using the standard value for the obliquity of the ecliptic (Table 12.1).

Satellite Ephemeris

The geocentric ephemeris of the Hipparcos satellite, $\mathbf{g}_0(T)$, was supplied by ESOC in the form of auxiliary Keplerian elements (describing a reference orbit) and 10 Chebyshev coefficients for each of the components in position and velocity, describing the deviations from the reference orbit over a given interval of time. Additionally, the full geocentric position and velocity vectors at a given reference time were supplied. Usually two to four intervals of such data were given per orbital period. Accuracies to which the various orbital parameters could be determined are given in Volume 2, Chapter 6.

To compute the position and velocity vectors at an arbitrary instant, the Keplerian elements were used to compute the reference vectors, to which were then added the corrections evaluated from the Chebyshev polynomials. The resulting data referred to the mean equator and equinox of the date. For subsequent combination with the Earth ephemeris the data were transformed to the equinox of J2000 by means of standard formulae for the precession (Table 5.1–5.2 in Murray 1983).

In the NDAC great-circle reductions, the satellite ephemeris was completely expressed as Chebyshev polynomials for the fraction of the orbit during which science data had been successfully obtained. The order of this representation (up to 10) depended on the length of the interval under consideration.

A comparison of the geocentric velocity components was made in 1991 by H. Schrijver using the FAST 'First Look' facility at SRON (Utrecht), and L. Lindegren using results of the NDAC great-circle reduction software. For three different observational frames in orbit 79, separated by 191 min and 1.5 min, the absolute differences in the computed equatorial components were found to be 0.043, 0.030 and 0.031 m s^{-1}, respectively. The maximum difference corresponds to 0.03 mas in stellar aberration. The result of this comparison was thus considered satisfactory.

12.2. Timing of the Observational Data

All observations on board Hipparcos, including the gyro readings, must be put on a single, continuous time scale with a well-defined relationship to the time scales used to describe the celestial phenomena. The requirement on the absolute timing of the data were derived from the most rapid variations in the calculated proper directions. For instance, the maximum acceleration of the satellite (at ~ 10 000 km altitude) was about 1.5 m s^{-2}, producing a maximum rate of change in stellar aberration of 1.0 mas s^{-1}. Thus, for the calculation of aberration the timing had to be accurate to better than ±100 ms in order not to introduce an error exceeding 0.1 mas. The most severe requirement stemmed however from the motions of minor planets, where the maximum geocentric angular velocity was approximately 20 mas s^{-1}, leading to a requirement of ±5 ms or better in the absolute timing.

The time scale used in all the astrometric reductions was Terrestrial Time, TT (equivalent to the former time scales TDT and ET). This is a continuous time scale with a simple relation to atomic time and suitable for describing celestial phenomena when an absolute accuracy of a few milliseconds is sufficient.

As described in Chapter 8, the timing of the data collected by the Hipparcos satellite was a combination of on-board computer regulation (driven by the on-board clock), and everything that happened between the emission of the data by the satellite and the time-tagging at the ground-station. These latter effects are fully described in Chapter 8; only the on-board timing is considered here, and only to the extent as noticed by the data reduction groups.

The scientific data and all auxiliary data were collected on-board in telemetry formats covering 32/3 s, or five observational frames. Most of the data accumulated did not, however, coincide exactly with the boundaries of the telemetry format: all gyro data (supplied 10 times per format) were shifted by 1.120 s, meaning that the first gyro data in a format referred in time to the last gyro integration interval of the preceding format. Similarly, the main detector (image dissector tube) and star mapper data were shifted, but in the case of the main detector data most of this shift was removed in the handling of the data at ESOC. The only shifts left in the data were those introduced by the on-board computer, which were of the order of 0.2 to 0.5 ms. The attitude reconstruction was represented such that the values for the angles and rates derived for an observational frame referred to the actual mid-time of the observational frame defined by the main detector samples, thus incorporating the 0.5 ms shift of the image dissector tube data.

A detailed comparison of timing calculations was made in 1991 between the Utrecht ‘First look’ reduction and special calculations by L. Lindegren, using NDAC data. The mid-times of two frames in orbit 429 and 696 were considered, and the calculations included the conversion of the time tag of the telemetry format from UTC to TT, application of the internal delay of the ground station, and of the propagation delay from satellite to antenna. The computed frame mid-times, expressed in TT at the satellite, differed by $-13\ \mu$s in one frame and by $+1\ \mu$s in the other. A further comparison with the frame mid-times calculated at RGO for the NDAC routine processing gave differences of +1.3 ms and −0.7 ms, but in all cases the results were considered satisfactory (see also Section 8.2 for further comments about some uncertainties in the ground-station delay times and the variations in the on-board clock, which were dealt with in different ways by the two consortia).

12.3. Coordinates for Stars and Solar System Objects

Following the terminology of Murray (1983), the proper direction to an object, as measured by a moving observer, is obtained by three successive transformations:

(1) the first is a translation of space-time coordinates from the adopted reference point (at the solar system barycentre) and epoch to the observer at the time of observation. In general this involves the space-time coordinates (in a given metric) of two specific events: the emission of light at the object, and its reception at the observer. For the observation of a solar system object these events are described by the ephemerides of the object and observer. For a stellar observation the transformation corresponds to the application of parallax and proper motion. In either case the result is the ‘coordinate

direction' to the object ($\bar{\mathbf{u}}$), expressed in the adopted metric. It should be noted that, in the context of General Relativity, the coordinate direction is a mathematical concept devoid of physical meaning, as it depends completely on the choice of metric. For the Hipparcos reductions, isotropic coordinates were used, usually assuming a spherically symmetric, heliocentric metric; the ephemerides described in the previous section were assumed to be expressed in such coordinates;

(2) the photon track from object to observer, when expressed in isotropic coordinates, is a hyperbola (if only light bending from the Sun is taken into account). The second transformation gives the direction of the photon track, as the light reaches the observer, relative to the 'natural frame' of the observer. This frame is a locally flat (Euclidean) coordinate system at rest with respect to the barycentre. The resulting 'natural direction' to the object ($\hat{\mathbf{u}}$) is what would be measured by a hypothetical stationary observer located at the same position as the real observer at the epoch of observation. The transformation from the coordinate direction to the natural direction corresponds to the application of gravitational light deflection, and again depends on the adopted metric. The resulting direction is however an observable entity, and therefore independent of the metric;

(3) the actual observer is moving relative to the natural frame, and the actually observed direction, or the 'proper direction' ($\mathbf{u}$), is obtained by a Lorentz transformation to the co-moving 'proper frame'. This corresponds to the application of stellar aberration.

These transformations are described below in the precise form that was used by NDAC. Equivalent representations were used by FAST and are described by Walter *et al.* (1986). All calculations were carried out on the equatorial direction cosines of the celestial coordinates, i.e. on unit vectors of the form:

$$\mathbf{u} = \begin{pmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{pmatrix} \qquad [12.1]$$

Isotropic Coordinate Direction

The catalogue data for stars refer to a specific epoch T_0, equal to J1991.25 for the final catalogue (J1990.0 and other epochs were also used in the reductions). The barycentric coordinate direction to a star at this epoch is:

$$\mathbf{u}_\mathrm{B}(T_0) = \begin{pmatrix} \cos\delta_0\cos\alpha_0 \\ \cos\delta_0\sin\alpha_0 \\ \sin\delta_0 \end{pmatrix} \qquad [12.2]$$

At the time of observation T, with the observer at barycentric coordinates $\mathbf{b}_0(T)$, the isotropic coordinate direction to the star is given by:

$$\bar{\mathbf{u}}(T) = \left\langle \mathbf{u}_\mathrm{B}(T_0) - \pi\mathbf{b}_0(T)A^{-1} + (T - T_0)\begin{pmatrix} -\sin\alpha_0 & -\sin\delta_0\cos\alpha_0 \\ \cos\alpha_0 & -\sin\delta_0\sin\alpha_0 \\ 0 & \cos\delta_0 \end{pmatrix}\begin{pmatrix} \mu_{\alpha*} \\ \mu_\delta \end{pmatrix} \right\rangle \qquad [12.3]$$

where π is the stellar parallax, $\mu_{\alpha*} = \mu_\alpha\cos\delta$, μ_δ the proper motion components, and A the astronomical unit (Table 12.1). In cases when parallax or proper motion information was not available, as in the initial reductions using the Hipparcos Input Catalogue, these values were assumed zero.

For a solar system object with geocentric ephemeris $\mathbf{g}(T)$, the isotropic coordinate direction is given by:

$$\bar{\mathbf{u}}(T) = \langle\, \mathbf{g}(T) - \mathbf{g}_0(T)\,\rangle \qquad [12.4]$$

Since planetary aberration was included in the geocentric ephemerides of solar system objects, the relevant argument of $\mathbf{g}$ is the time of observation, T, and not the time of light emission, $T - |\mathbf{g} - \mathbf{g}_0|c^{-1}$.

Natural Direction

The transformation from the isotropic coordinate direction to the natural direction took into account the light bending by the Sun and (in NDAC only) by the Earth; for solar system objects, their finite distances from Hipparcos were taken into account. The relevant transformation in the heliocentric isotropic metric is given by Equation 2.5.5 in Murray (1983). Adding an extra term for the deflection by the Earth (which may amount to a few tenths of a milliarcsec) and re-writing for computational efficiency (see Section 12.4) gives the following formula for the natural direction:

$$\hat{\mathbf{u}} = \left\langle\, \bar{\mathbf{u}} + \mathbf{h}_0 \frac{2GS}{c^2 h_0 (q_S h_0 + \bar{\mathbf{u}}'\mathbf{h}_0)} + \mathbf{g}_0 \frac{2GE}{c^2 g_0 (q_E g_0 + \bar{\mathbf{u}}'\mathbf{g}_0)} \right\rangle \qquad [12.5]$$

where $\mathbf{h}_0$ and $\mathbf{g}_0$ are the heliocentric and geocentric positions of Hipparcos, and h_0, g_0 the corresponding distances. The heliocentric and geocentric gravitational constants are given by GS and GE respectively (Table 12.1). The scalars q_S and q_E were set to one for stars, while for objects in the solar system they were computed according to the distance s from Hipparcos:

$$\begin{aligned} q_S &= |\bar{\mathbf{u}} + \mathbf{h}_0 s^{-1}| + h_0 s^{-1} \\ q_E &= |\bar{\mathbf{u}} + \mathbf{g}_0 s^{-1}| + g_0 s^{-1} \end{aligned} \qquad [12.6]$$

Proper Direction

The proper direction to a star or solar system object was obtained by a Lorentz transformation depending on the velocity $\mathbf{V}$ of the observer; see e.g. Equation 2.5.8 in Murray (1983). The velocity was taken to be the sum of the geocentric velocity, $\mathrm{d}\mathbf{g}_0/\mathrm{d}T$, computed from the satellite ephemeris, and the barycentric velocity of the Earth, $\mathrm{d}\mathbf{b}_E/\mathrm{d}T$, computed from the Earth ephemeris. With $e = (c^2 - V^2)^{1/2}$ the proper direction can be written (see Section 12.4):

$$\mathbf{u} = \left\langle \hat{\mathbf{u}} + \mathbf{V}\left[1 + \mathbf{V}'\hat{\mathbf{u}}(c+e)^{-1}\right] e^{-1} \right\rangle \qquad [12.7]$$

Comparison of FAST, NDAC, and TDAC Calculations

The participants in this comparison were requested to provide all proper directions computed by them for a few stars in a given orbit, i.e. for all frames in all transits for the main mission reduction, and for all the star mapper transits for the Tycho reduction. The selected orbit was number 79 (9–10 December 1989), and the selected stars were HIP 7680, 7708, 67186, and 67362.

Data were received in February–April 1992 from the NDAC main data reduction (supplied by C. Petersen), the TDAC reduction (supplied by U. Bastian), and the FAST main reduction at CERGA (supplied by J. Kovalevsky). Also available were the results from the Utrecht 'First Look' reduction (SRON).

The data were put on a common frame numbering system, using the timing data provided by the participants, and the position data were transformed to a common coordinate system. The equatorial or ecliptic system would be natural, but presented a practical problem: in the CERGA and SRON data reductions, only the component parallel to the reference great circle, influencing directly the abscissae, was computed with full precision. For the perpendicular component, the change in position during the transit was not computed, nor the contribution from gravitational deflection. This leads to errors not greater than 10 mas in the ordinate. So a useful comparison was only possible in the spherical coordinates (abscissa, ordinate) defined relative to the reference great circle used by CERGA for this orbit. The proper directions were therefore transformed into this coordinate system.

The Tycho data were only compared with NDAC, and the differences could be expressed directly in the equatorial angles, as both sources provided the proper directions to full accuracy in both coordinates. The TDAC data were given for the star mapper transit times, which fell beyond the interval covered by the main mission data for the same field transit. Linear extrapolation of the main mission data to the star mapper transit times was therefore used to enable comparison with the Tycho data.

Each participant used their own catalogue of astrometric parameters for computing the proper directions. These catalogues were all different, reflecting the various stages of catalogue updating or iteration at the different establishments. The components of these differences in abscissa and ordinate were computed and subtracted from the comparisons. The results are summarised in Table 12.3 for the main mission comparison (CERGA, NDAC, SRON) and in Table 12.4 for the TDAC/NDAC comparison.

The comparison showed good agreement between the proper directions computed by the four participants, with the exception of the known inaccuracy in the transverse great-circle direction for CERGA and SRON. Especially the average NDAC–CERGA differences in abscissa, of the highest relevance for the actual reductions, were gratifyingly small (< 0.05 mas). At the same time, this result validated the computation of the orbital velocity of the satellite, which was responsible for most of the evolution of the apparent positions observed during the great-circle set.

12.4. Formulae for Gravitational Deflection and Aberration

Equations 12.5–12.7 deviate somewhat from the standard formulae given, for example, in the *Explanatory Supplement to the Astronomical Almanac* (Seidelmann 1992). For completeness, they are here derived from the corresponding expressions given in Murray (1983).

Natural Direction (Gravitational Deflection)

Equation 2.5.5 in Murray (1983) gives the natural direction in terms of the isotropic, heliocentric coordinates of the object and observer. With $\mathbf{h}$ and $\mathbf{h}_0$ denoting the heliocentric positions of the object and Hipparcos (at distances h and h_0 from the Sun), the coordinate direction to the object is $\bar{\mathbf{u}} = \mathbf{s}s^{-1}$, where $\mathbf{s} = \mathbf{h} - \mathbf{h}_0$. Murray's Equation 2.5.5 can then be written:

$$\hat{\mathbf{u}} = \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0}(hh_0 + \mathbf{h}'\mathbf{h}_0)^{-1}(\mathbf{h} \times \mathbf{h}_0) \times \bar{\mathbf{u}} \qquad [12.8]$$

Substituting $\mathbf{h} = \mathbf{h}_0 + \bar{\mathbf{u}}s$ gives:

$$\hat{\mathbf{u}} = \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0(qh_0 + \bar{\mathbf{u}}'\mathbf{h}_0)}(\bar{\mathbf{u}} \times \mathbf{h}_0) \times \bar{\mathbf{u}} \qquad [12.9]$$

where $q = (h + h_0)/s = |\bar{\mathbf{u}} + \mathbf{h}_0 s^{-1}| + h_0 s^{-1}$. The vector $(\bar{\mathbf{u}} \times \mathbf{h}_0) \times \bar{\mathbf{u}} = \mathbf{h}_0 - \bar{\mathbf{u}}\bar{\mathbf{u}}'\mathbf{h}_0$ is the projection of $\mathbf{h}_0$ in the plane normal to $\bar{\mathbf{u}}$. An equivalent form of Equation 12.9, to first order in the factor $2GS/c^2 s$, is therefore:

$$\hat{\mathbf{u}} = \left\langle \bar{\mathbf{u}} + \frac{2GS}{c^2 h_0(qh_0 + \bar{\mathbf{u}}'\mathbf{h}_0)}\mathbf{h}_0 \right\rangle \qquad [12.10]$$

Equation 12.5 was obtained from this form by adding another term representing the deflection by the Earth.

Proper Direction (Stellar Aberration)

Equation 2.5.8 in Murray (1983) gives the following transformation from the natural direction $\hat{\mathbf{u}}$ to the proper direction $\mathbf{u}$:

$$\mathbf{u} = \frac{[\mathbf{U} + (\beta - 1)\mathbf{v}\mathbf{v}'](\hat{\mathbf{u}} + c^{-1}\mathbf{V})}{\beta(1 + c^{-1}\hat{\mathbf{u}}'\mathbf{V})} \qquad [12.11]$$

$\mathbf{U}$ is the unit tensor, $\beta = (1 - V^2/c^2)^{-1/2}$ and $\mathbf{v} = \langle \mathbf{V} \rangle = \mathbf{V}V^{-1}$, where $\mathbf{V}$ is the velocity of the observer in the natural frame. The denominator in this equation is just a normalising factor making $\mathbf{u}$ a unit vector. An equivalent form is therefore:

$$\begin{aligned}\mathbf{u} &= \left\langle \hat{\mathbf{u}} + \frac{1}{c}\mathbf{V} + (\beta - 1)\mathbf{v}\mathbf{v}'\hat{\mathbf{u}} + \frac{\beta - 1}{c}\mathbf{V} \right\rangle \\ &= \left\langle \hat{\mathbf{u}} + \frac{\beta}{c}\mathbf{V} + (\beta - 1)\mathbf{v}\mathbf{v}'\hat{\mathbf{u}} \right\rangle \qquad [12.12]\end{aligned}$$

since $\mathbf{v}\mathbf{v}'\mathbf{V} = \mathbf{V}$. For numerical accuracy the factor $(\beta - 1)$, which is of order $(V/c)^2$, should be written in a form which avoids taking the difference of two nearly equal numbers. Introducing $e = (c^2 - V^2)^{1/2}$ gives $\beta = c/e$ and:

$$(\beta - 1)\mathbf{v}\mathbf{v}' = \frac{c - e}{e}\mathbf{v}\mathbf{v}' = \frac{c^2 - e^2}{e(c + e)}\mathbf{v}\mathbf{v}' = \frac{V^2}{e(c + e)}\mathbf{v}\mathbf{v}' = \frac{1}{e(c + e)}\mathbf{V}\mathbf{V}' \qquad [12.13]$$

Inserting this into Equation 12.12 gives the aberration formula in the form implemented by NDAC, i.e. Equation 12.7.

L. Lindegren, F. van Leeuwen, J. Kovalevsky

Table 12.1. Fundamental constants used in the coordinate calculations.

Constant	Meaning	Value	Unit
c	Speed of light	299 792 458	m s^{-1}
ϵ	Obliquity of ecliptic (J2000.0)	23° 26′ 21.448″	
A	Astronomical unit	$1.495\,978\,701 \times 10^{11}$	m
GS	Heliocentric gravitational constant	$1.327\,124\,38 \times 10^{20}$	$m^3 s^{-2}$
GE	Geocentric gravitational constant	$3.986\,005 \times 10^{14}$	$m^3 s^{-2}$

Table 12.2. A tabulation of the Earth ephemeris used in the reductions, at intervals of 10 days over the mission. T is the time in days from JD 2 440 000.0(TT). X, Y, Z are the barycentric equatorial coordinates of the Earth in km. The time derivatives give the barycentric velocity components in m s^{-1}. The reference system is the mean equator and equinox of J2000.

T	X	Y	Z	dX/dT	dY/dT	dZ/dT
7800.0	+148 357 230	+17 858 435	+7 740 978	−4 340.289	+27 006.475	+11 710.082
7810.0	+142 430 117	+40 807 475	+17 690 769	−9 345.037	+25 981.084	+11 263.454
7820.0	+132 286 866	+62 538 103	+27 112 230	−14 088.914	+24 202.245	+10 494.289
7830.0	+118 180 524	+82 412 426	+35 729 900	−18 490.126	+21 676.876	+9 398.543
7840.0	+100 496 735	+99 792 639	+43 264 714	−22 337.008	+18 451.134	+7 998.782
7850.0	+79 771 704	+114 141 069	+49 485 893	−25 529.354	+14 677.488	+6 364.766
7860.0	+56 592 734	+125 009 555	+54 198 420	−27 987.382	+10 405.628	+4 510.779
7870.0	+31 667 938	+132 020 751	+57 237 269	−29 555.217	+5 785.373	+2 507.810
7880.0	+5 774 488	+134 962 007	+58 512 945	−30 238.496	+1 000.475	+ 434.460
7890.0	−20 316 030	+133 725 584	+57 976 493	−29 992.966	−3 861.158	−1 675.574
7900.0	−45 771 911	+128 328 148	+55 635 417	−28 779.737	−8 591.348	−3 724.810
7910.0	−69 805 070	+118 962 234	+51 575 137	−26 714.560	−13 038.323	−5 652.977
7920.0	−91 692 277	+105 906 361	+45 913 574	−23 811.880	−17 104.555	−7 417.693
7930.0	−110 735 936	+89 573 280	+38 831 561	−20 166.429	−20 596.142	−8 929.190
7940.0	−126 380 730	+70 499 363	+30 561 995	−15 959.997	−23 448.356	−10 167.146
7950.0	−138 171 168	+49 258 803	+21 351 515	−11 259.773	−25 588.406	−11 095.610
7960.0	−145 750 953	+26 524 143	+11 494 492	−6 258.045	−26 901.425	−11 662.873
7970.0	−148 949 966	+2 998 879	+1 294 552	−1 131.510	−27 424.461	−11 891.661
7980.0	−147 695 299	−20 629 468	−8 951 132	+4 026.301	−27 127.049	−11 761.883
7990.0	−142 050 756	−43 636 197	−18 925 742	+8 991.844	−26 003.389	−11 273.844
8000.0	−132 245 224	−65 358 483	−28 344 425	+13 651.112	−24 163.193	−10 477.951
8010.0	−118 574 862	−85 186 112	−36 941 795	+17 909.061	−21 616.311	−9 371.706
8020.0	−101 472 152	−102 535 520	−44 463 462	+21 576.873	−18 458.839	−8 003.311
8030.0	−81 468 107	−116 946 860	−50 712 665	+24 623.183	−14 825.097	−6 429.014
8040.0	−59 124 581	−128 026 080	−55 516 337	+26 964.488	−10 757.019	−4 662.900
8050.0	−35 109 793	−135 459 596	−58 738 959	+28 494.080	−6 420.298	−2 784.338
8060.0	−10 108 138	−139 077 867	−60 308 671	+29 250.809	−1 933.605	− 838.943
8070.0	+15 194 245	−138 774 851	−60 176 742	+29 170.845	+2 632.871	+1 142.574
8080.0	+40 057 155	−134 562 658	−58 350 505	+28 252.521	+7 088.607	+3 072.307
8090.0	+63 798 900	−126 581 233	−54 890 675	+26 579.069	+11 352.343	+4 922.386

Table 12.2. (Continued).

T	X	Y	Z	dX/dT	dY/dT	dZ/dT
8100.0	+85 756 907	−115 026 886	−49 880 178	+24 120.588	+15 327.328	+6 646.094
8110.0	+105 283 302	−100 229 023	−43 464 706	+20 978.678	+18 845.667	+8 169.741
8120.0	+121 842 972	−82 602 855	−35 822 695	+17 256.585	+21 870.223	+9 483.374
8130.0	+134 939 519	−62 612 062	−27 154 469	+12 980.005	+24 288.787	+10 530.877
8140.0	+144 166 076	−40 837 230	−17 714 259	+8 329.699	+25 998.369	+11 271.291
8150.0	+149 256 863	−17 886 770	−7 763 237	+3 410.677	+27 005.717	+11 710.190
8160.0	+150 008 266	+5 595 999	+2 418 607	−1 680.048	+27 212.701	+11 797.965
8170.0	+146 368 346	+28 903 249	+12 523 202	−6 725.473	+26 611.644	+11 537.894
8180.0	+138 424 723	+51 365 015	+22 262 708	−11 634.283	+25 251.192	+10 949.107
8190.0	+126 349 147	+72 304 171	+31 341 146	−16 248.782	+23 089.754	+10 009.933
8200.0	+110 489 942	+91 061 509	+39 473 275	−20 373.448	+20 225.844	+8 769.958
8210.0	+91 298 683	+107 077 962	+46 418 294	−23 948.886	+16 744.488	+7 260.089
8220.0	+69 315 205	+119 830 181	+51 946 583	−26 803.691	+12 693.260	+5 502.232
8230.0	+45 223 723	+128 902 071	+55 879 810	−28 826.905	+8 257.635	+3 581.341
8240.0	+19 740 740	+134 017 693	+58 098 207	−30 010.703	+3 542.191	+1 535.126
8250.0	−6 357 215	+134 979 946	+58 514 398	−30 237.037	−1 320.044	− 573.178
8260.0	−32 237 247	+131 759 277	+57 118 355	−29 525.043	−6 111.978	−2 648.998
8270.0	−57 122 429	+124 462 921	+53 954 730	−27 926.074	−10 740.416	−4 658.027
8280.0	−80 231 740	+113 296 608	+49 112 356	−25 424.842	−15 032.619	−6 517.873
8290.0	−100 840 262	+98 635 933	+42 756 484	−22 169.469	−18 816.245	−8 157.761
8300.0	−118 349 405	+80 940 705	+35 083 668	−18 252.767	−22 040.940	−9 557.849
8310.0	−132 214 681	+60 754 173	+26 330 749	−13 765.299	−24 556.352	−10 646.386
8320.0	−142 038 310	+38 732 354	+16 783 218	−8 928.813	−26 295.797	−11 401.429
8330.0	−147 569 835	+15 536 252	+6 725 043	−3 841.726	−27 262.550	−11 821.545
8340.0	−148 648 921	−8 128 956	−3 535 581	+1 336.569	−27 373.298	−11 867.371
8350.0	−145 298 572	−31 534 580	−13 683 440	+6 392.449	−26 681.743	−11 569.403
8360.0	−137 657 769	−54 016 616	−23 432 038	+11 251.638	−25 230.499	−10 939.806
8370.0	−125 963 812	−74 910 751	−32 490 694	+15 735.107	−23 018.200	−9 979.269
8380.0	−110 616 827	−93 616 238	−40 601 171	+19 704.413	−20 187.243	−8 753.903
8390.0	−92 073 143	−109 632 912	−47 546 253	+23 118.059	−16 796.139	−7 281.989
8400.0	−70 876 147	−122 497 430	−53 123 244	+25 819.795	−12 919.844	−5 601.453
8410.0	−47 670 759	−131 871 695	−57 188 355	+27 775.122	−8 734.996	−3 788.468
8420.0	−23 098 703	−137 519 036	−59 637 031	+28 968.432	−4 300.502	−1 863.486
8430.0	+2 134 509	−139 269 173	−60 395 237	+29 300.023	+ 248.515	+ 107.329
8440.0	+27 302 867	−137 103 618	−59 457 163	+28 831.358	+4 752.681	+2 059.887
8450.0	+51 726 456	−131 084 090	−56 846 810	+27 564.573	+9 156.077	+3 971.150
8460.0	+74 696 530	−121 361 504	−52 631 136	+25 483.543	+13 289.575	+5 761.081
8470.0	+95 566 050	−108 228 680	−46 937 834	+22 716.625	+17 045.571	+7 390.496
8480.0	+113 759 001	−92 029 008	−39 913 267	+19 283.827	+20 368.964	+8 832.359
8490.0	+128 719 443	−73 206 186	−31 752 480	+15 267.487	+23 095.599	+10 012.492
8500.0	+140 020 910	−52 302 703	−22 689 683	+10 828.388	+25 186.436	+10 920.891
8510.0	+147 320 330	−29 880 070	−12 967 022	+6 012.807	+26 589.561	+11 528.647
8520.0	+150 353 454	−6 588 572	−2 869 148	+ 992.927	+27 194.239	+11 789.702
8530.0	+149 024 751	+16 892 384	+7 311 664	−4 071.455	+27 035.105	+11 722.852
8540.0	+143 324 817	+39 898 177	+17 286 815	−9 100.608	+26 077.504	+11 305.856
8550.0	+133 374 933	+61 719 208	+26 747 008	−13 873.176	+24 308.222	+10 538.945
8560.0	+119 462 652	+81 700 782	+35 411 061	−18 265.108	+21 831.478	+9 466.613
8570.0	+101 950 256	+99 239 546	+43 015 326	−22 172.007	+18 652.508	+8 086.103
8580.0	+81 349 265	+113 759 444	+49 310 129	−25 392.139	+14 874.203	+6 449.372
8590.0	+58 284 891	+124 816 410	+54 104 944	−27 873.953	+10 650.351	+4 618.188

Table 12.2. (Continued).

T	X	Y	Z	dX/dT	dY/dT	dZ/dT
8600.0	+33 423 912	+132 049 230	+57 240 318	−29 520.315	+6 040.855	+2 617.950
8610.0	+7 547 257	+135 200 297	+58 606 317	−30 223.404	+1 239.997	+ 538.624
8620.0	−18 546 133	+134 184 993	+58 166 798	−30 029.030	−3 592.284	−1 557.825
8630.0	−44 068 206	+129 011 773	+55 922 959	−28 888.245	−8 352.287	−3 622.205
8640.0	−68 199 485	+119 837 477	+51 945 514	−26 830.948	−12 820.207	−5 557.236
8650.0	−90 209 937	+106 974 125	+46 368 576	−23 989.873	−16 886.615	−7 322.524
8660.0	−109 432 022	+90 805 123	+39 357 291	−20 386.570	−20 439.119	−8 862.092
8670.0	−125 262 447	+71 853 573	+31 141 079	−16 176.897	−23 310.222	−10 105.724
8680.0	−137 262 031	+50 724 816	+21 979 952	−11 529.203	−25 479.876	−11 048.582
8690.0	−145 080 682	+28 051 311	+12 148 827	−6 525.942	−26 863.952	−11 646.967
8700.0	−148 501 427	+4 550 786	+1 960 401	−1 389.945	−27 399.489	−11 879.451
8710.0	−147 484 435	−19 072 576	−8 282 736	+3 739.335	−27 152.253	−11 773.510
8720.0	−142 073 628	−42 131 273	−18 280 348	+8 747.065	−26 085.895	−11 309.053
8730.0	−132 469 575	−63 926 276	−27 729 547	+13 416.836	−24 251.852	−10 515.522
8740.0	−119 005 090	−83 851 696	−36 369 503	+17 678.118	−21 762.463	−9 436.123
8750.0	−102 072 865	−101 345 526	−43 953 818	+21 411.863	−18 631.545	−8 076.953
8760.0	−82 204 073	−115 903 546	−50 265 733	+24 467.439	−15 000.327	−6 504.836
8770.0	−59 986 298	−127 155 058	−55 144 752	+26 844.652	−10 982.183	−4 761.361
8780.0	−36 041 162	−134 784 257	−58 451 718	+28 441.480	−6 636.487	−2 876.672
8790.0	−11 080 616	−138 585 358	−60 100 224	+29 207.017	−2 151.048	− 933.817
8800.0	+14 204 317	−138 485 336	−60 057 112	+29 188.408	+2 387.717	+1 036.167
8810.0	+39 110 530	−134 469 949	−58 315 331	+28 319.406	+6 881.390	+2 983.540
8820.0	+62 912 359	−126 663 547	−54 931 445	+26 656.315	+11 144.827	+4 831.253
8830.0	+84 965 033	−115 292 234	−50 000 888	+24 268.960	+15 125.301	+6 559.223
8840.0	+104 634 311	−100 643 397	−43 649 094	+21 147.289	+18 697.280	+8 105.881
8850.0	+121 340 756	−83 141 819	−36 061 639	+17 438.267	+21 724.263	+9 418.838
8860.0	+134 621 727	−63 266 083	−27 443 364	+13 220.696	+24 183.632	+10 486.522
8870.0	+144 054 532	−41 554 021	−18 029 681	+8 558.925	+25 947.402	+11 249.001
8880.0	+149 343 923	−18 644 464	−8 097 170	+3 654.397	+26 962.757	+11 690.842
8890.0	+150 322 985	+4 825 459	+2 079 615	−1 410.381	+27 235.603	+11 809.123
8900.0	+146 899 575	+28 175 443	+12 202 984	−6 494.247	+26 675.305	+11 564.719
8910.0	+139 156 166	+50 695 285	+21 967 017	−11 391.622	+25 330.504	+10 983.737
8920.0	+127 289 553	+71 733 218	+31 089 181	−16 024.224	+23 239.327	+10 075.597
8930.0	+111 598 056	+90 631 785	+39 282 316	−20 203.361	+20 392.175	+8 841.003
8940.0	+92 553 535	+106 795 727	+46 291 112	−23 777.067	+16 932.958	+7 342.982
8950.0	+70 698 914	+119 736 511	+51 902 120	−26 689.652	+12 932.861	+5 606.350
8960.0	+46 676 994	+129 013 224	+55 923 587	−28 767.722	+8 485.363	+3 679.198
8970.0	+21 242 749	+134 330 125	+58 229 732	−29 964.724	+3 792.747	+1 645.398
8980.0	−4 847 726	+135 520 693	+58 745 638	−30 270.118	−1 054.798	− 458.493
8990.0	−30 779 383	+132 513 157	+57 441 364	−29 596.585	−5 883.382	−2 550.042
9000.0	−55 732 174	+125 418 207	+54 366 088	−28 025.351	−10 497.032	−4 551.012
9010.0	−78 961 203	+114 456 098	+49 612 556	−25 598.979	−14 815.787	−6 424.572
9020.0	−99 729 713	+99 957 921	+43 326 761	−22 353.763	−18 648.133	−8 084.066
9030.0	−117 406 885	+82 409 179	+35 718 691	−18 472.096	−21 873.158	−9 484.184
9040.0	−131 486 498	+62 346 025	+27 019 140	−14 031.872	−24 447.531	−10 599.995
9050.0	−141 533 615	+40 393 426	+17 501 776	−9 176.208	−26 230.366	−11 371.529
9060.0	−147 288 604	+17 250 811	+7 467 842	−4 120.697	−27 214.353	−11 800.423
9070.0	−148 617 442	−6 402 958	−2 788 206	+1 051.827	−27 397.918	−11 878.435
9080.0	−145 493 784	−29 845 973	−12 951 501	+6 145.538	−26 730.569	−11 588.870
9090.0	−138 073 850	−52 377 985	−22 721 155	+10 986.257	−25 309.858	−10 974.590
9100.0	−126 599 628	−73 370 974	−31 823 046	+15 506.337	−23 162.430	−10 041.396

Table 12.3. Comparison of proper directions computed by CERGA, NDAC and SRON for the main mission reductions. All comparisons refer to orbit number 79. The third and fourth columns give the number of observational frames compared, and the time span of the comparison in minutes. Subsequent columns give the average, rms and extreme values of the differences in abscissa and ordinate (the rms value being the dispersion about the average difference).

Star (HIP)	Sources compared	*N*	Span (min)	Abscissa differences (mas) ave	rms	min	max	Ordinate differences (mas) ave	rms	min	max
	CERGA–SRON	59	277	+0.14	0.069	+0.02	+0.28	−0.12	0.134	−0.31	+0.07
7680	NDAC–CERGA	51	277	+0.04	0.076	−0.10	+0.19	−1.13	0.685	−2.07	+0.76
	SRON–NDAC	51	277	−0.19	0.073	−0.31	−0.11	+1.22	0.641	−0.56	+2.11
	CERGA–SRON	59	277	+0.13	0.067	+0.01	+0.29	−0.07	0.102	−0.20	+0.08
7708	NDAC–CERGA	56	277	+0.04	0.081	−0.12	+0.21	−0.92	0.712	−1.93	+0.83
	SRON–NDAC	56	277	−0.17	0.086	−0.32	−0.07	+0.99	0.675	−0.66	+2.07
	CERGA–SRON	69	384	+0.00	0.057	−0.09	+0.10	−0.16	0.155	−0.42	+0.02
67186	NDAC–CERGA	69	384	+0.03	0.061	−0.07	+0.14	−5.69	0.937	−6.68	−2.29
	SRON–NDAC	78	405	−0.03	0.041	−0.10	+0.05	+5.71	1.066	+1.92	+7.11
	CERGA–SRON	64	384	+0.09	0.088	−0.07	+0.30	−0.07	0.114	−0.23	+0.09
67362	NDAC–CERGA	54	384	+0.01	0.061	−0.10	+0.14	−4.41	0.996	−5.55	−1.06
	SRON–NDAC	63	405	−0.07	0.079	−0.25	+0.04	+4.34	1.117	+1.02	+5.85

Table 12.4. Comparison of proper directions computed by TDAC and NDAC. All comparisons were made for orbit number 79. The third and fourth columns give the number of star mapper transits used in the comparison, and the time span of the comparison in minutes. Subsequent columns give the average, rms and extreme values of the differences in abscissa and ordinate (the rms value being the dispersion about the average difference).

Star (HIP)	Sources compared	*N*	Span (min)	Abscissa differences (mas) ave	rms	min	max	Ordinate differences (mas) ave	rms	min	max
7680	TDAC–NDAC	10	256	−0.0	0.07	−0.1	+0.1	−0.0	0.09	−0.1	+0.1
7708	TDAC–NDAC	10	256	−0.0	0.11	−0.1	+0.2	+0.0	0.08	−0.1	+0.1
67186	TDAC–NDAC	12	277	+0.1	0.09	+0.0	+0.2	−0.0	0.08	−0.1	+0.1
67362	TDAC–NDAC	8	149	+0.1	0.07	+0.0	+0.2	−0.1	0.05	−0.1	+0.0